1964

Problem Solving Processes Amd Grades in Mathematics

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PROBLEM SOLVING PROCESSES

AND

GRADES IN MATHEMATICS

by

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A Thesis Submitted to the Faculty of the Graduate School
of Loyola University in Partial Fulfillment
of the Requirements for the Degree
of
Master of Arts

February

1964

Table of Contents

Chapter		Page
1.	Introduction................................	1
11.	Review of the Related Literature..................	3
111.	Procedure..................................	6
A. Purpose................................	6
B. Population: Description of the Sample...........	6
C. Description of the Problems Used...............	7
D. Scoring Method...........................	8
1. Number of Questions Asked.................	8
2. Correct Solution.........................	9
3. Group Norms............................	9
4. Ellipsoids and Parallelograms.............	10
5. Schemata Norms..........................	11
6. Plateaux...............................	11
7. Convex Sets............................	12
1V.	Results....................................	13
A. Number of Questions.......................	13
B. Correct Solutions.........................	13
C. Group Norms - In Terms of What the Group Did....	14
D. Ellipsoids and Parallelograms - How much the two groups agree or disagree in terms of the questions that they ask................................	16
E. Schemata Norms - In terms of the logical structure of the Problem............................	21

Chapter Page

F. Plateaux- Analysis of the Performance Curve.... 21

G. Convex Sets........ 23

V. Summary.............................. 30

VI. Bibliography........................... 33

VII. Appendix............................. 36

List of Figures

Figure		Page
1.	Ellipsoids and Parrallelograms for Problem 35A..............	17
2.	Ellipsoids and Parallelograms for Problem 35B...............	18
3.	Ellipsoids and Parrallelograms for Problem 39A..............	19
4.	Ellipsoids and Parrallelograms for Problem 39B..............	20
5.	Performance Curves for a Subject of the low group and a Subject of the high group (Schemata Norms) for Problem 35A and Problem 35B................................	22
6.	Convex Set (Schemata Norms) for Problem 35A.................	24a-b
7.	Convex Set (Schemata Norms) for Problem 35B.................	25a-b
8.	Convex Set (Group Norms) for Problem 35A....................	26a-b
9.	Convex Set (Group Norms) for Problem 35B....................	27
10.	Convex Set (Group Norms) for Problem 39A....................	28
11.	Convex Set (Group Norms) for Problem 39B....................	29

List of Tables

Table Number		Page
I.	Means of the Grades and Means and Standard Deviations Used in Selection of the Group...........	7
II.	Mean Number of Questions for Both Groups on Each of the Four Problems.........................	13
III.	Chi-Square Analysis of Correct and Incorrect Solutions.......................................	14
IVa.	Mean Utility Indices..............................	15
IV.	Mean Scores Based on Group Norms..................	15
V.	The Ratios Between the Ellipsoids and Parrallelograms and the Results of the Test of Significance..............................	16
VI.	Mean Scores According to the Schemata Norms.........	21
VII.	Mean Length of the Plateaux.........................	23

VITA

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ACKNOWLEDGEMENTS

The author is deeply thankful to Dr. H. J. A. Rimoldi, Mr. Gary Burger and Dr. Hermelinda M. Fogliatto for the kind help they gave in the completion of this research.

The author also wishes to thank Mrs. Catherine Sheehan and Mrs. Mary Jo Lutzow for their indispensable assistance in the presentation of this study.

This study was supported by the National Science Foundation, Project No. G-19844. Principal Investigator, H. J. A. Rimoldi, M.D., Ph. D.

CHAPTER I

INTRODUCTION

The objective of the present thesis is to find out what kind of relationship exists between processes involved in the solution of mathematical problems and corresponding grades in mathematics and science in high-school students.

This study has some features in common with the traditional work done in the areas of intelligence and problem solving. Intelligence and ability to solve problems have often been regarded as closely related with grades in school. However, the relation between these is not well known. Previous studies have focused upon correlational techniques to examine this relationship. The present study will focus upon the relationship between problem solving processes (rather than correct answers) and grades in school, utilizing the method developed by H. J. A. Rimoldi and associates at the Loyola Psychometric Laboratory.

The present study does not attempt to give a final word concerning the proper technique to use in order to investigate the processes involved in problem solving. In fact, as one reads this study, it is possible to see different avenues leading to further research. The history of the technique used here is illustrated in the review of the literature. A subject is asked to solve a problem. There are a number of questions (usually ranging from ten to twenty) which he can ask in order to gain the necessary information to solve the problem. The approach of the subject to problem solving can then be characterized in terms of the particular questions he selects and the particular order in which he selects them. This specific approach assumes that the performance of the subjects reflects thought processes as they develop. This study differs from similar traditional studies in that processes rather than products (final answers) is the focus of attention and in that the technique used to explore these processes is different.

This technique concerns itself with the study of the process which formerly could only be approached by such means as introspection and "thinking aloud". Moreover, it has the added advantages of objectivity and quantification.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

Waters (1928) investigated the effect of instruction upon ideational learning. He uses different types of instruction in order to discover which was the best type.

Doyle (1933) studied the similarities and differences between inductive discovery and trial and error processes in human learning. He did this by subjecting different groups of subjects to problem situations of varied complexity. After solving only one problem, the subject was able to discover the principle involved. The results showed clear distinction between the process of inductive discovery and the process of trial and error.

Dunker (1945) conducted a study to discover the processes involved in problem solving. His monograph has become a classical study in this field for the purpose of investigating thought process, he used the method of "thinking out loud". In solving the problems the subjects had to analyze what was given in the problem and what was required for the solution. The process of solving a problem consisted in the generation and testing of hypotheses. The role of past experience was also evaluated.

Bloom and Broder (1950) described the difficulties of attempting to discover the nature of mental processes. They constructed test situations such that a different method of attack would lead to different situations. They classified the students as successful or unsuccessful according to their aptitude scores and marks on comprehensive examinations. The students were asked to think aloud while they were solving the problems. In collecting data, complete notes were taken on everything that the subject said or did. The successful problem solver showed greater ability to understand the nature of the problem and to attack it in its own terms. The unsuccessful problem solvers showed lack of comprehension of direction and often presented solutions of a problem other than the one expected. "The nonsuccessful problem solvers started the problem with no apparent plan for solution. They jumped from one part of the problem to another, giving insufficient consideration to any one part to enable them to find a point of departure. They were easily side-tracked by external considerations, and their thoughts would go off on a tangent, coming back to the problem only with considerable difficulty".

Tate, Stanier and Harootunian (1959) classified students as good and poor problem solvers using as criteria their performance in a battery of tests, one of which was the "thought problems", a test that was specially prepared for that study. They concluded that the "good problem solvers are significantly better than the poor in nearly all tests where quality of response, accuracy, or judgement is required; and that, without exception, the more complex the task and the more restricted the requirements, the greater their superiority".

Bryan, Glaser, Damrin and Gardner analysed trouble-shooting ability in electronics. Their technique (tab-item technique) starts from different assumptions than the one used in this study.

The technique devised by Rimoldi: (1955) was first used to study diagnostic ability in medical students. The Loyola Psychometric Laboratory has published a series of studies which made use of this technique (Rimoldi, 1960, Rimoldi, 1961, Rimoldi and Haley, 1961). A final report by Rimoldi, Haley, Fogliatto (1962) summarized the whole work.

As mentioned earlier, this latter technique has been applied to areas other than medical diagnostics by Rimoldi and associates. Gunn (1961) applied this to appraise the personality of the person. In several papers (11,12,13,18,20,23) Rimoldi applied the technique to the study of processes involved in reasoning and problem solving. Rimoldi and Majewska (1961) presented a research proposal for the study of decision processes involved in mathematical thinking. Tabor (1959) used this technique for the processes involved in interpretation of the Rorschach. Rimoldi and Devane (1961) used this approach to study the effect of training in high-school students. This was again used to compare the effects of training in high-school children with those in college (Rimoldi, Fogliatto, Haley, Reyes, Erdmann and Zacharia, 1962).

All the above studies deal with the evaluation of the performances of the subjects in terms of group norms. Riedel (1963), Fogliatto (1963), studied the performance of the subjects in terms of schemata norms as well. This has been described by Rimoldi, Fogliatto, Haley, and Erdmann (1963).

The present research aims at comparing the results derived from schemata norms as well as group norms.

CHAPTER III

PROCEDURE

A) Purpose

The purpose of this study is to analyze and compare the problem solving processes of two groups of high school students. These two groups are defined in terms of their mathematical ability as measured by grades (plane geometry, physics, general science and elementary algebra). The study hopes to throw light on the relationship between grades and mental processes in problem solving. Only a passing reference will be made to the actual solution of the problem, this not being the primary purpose of this study.

B) Population

The subjects used for this study were obtained from two Chicago-area Catholic high schools. The problems to be analyzed were given to the entire junior class of each school. The particular sample used for this study was drawn from one of these schools (St. Rita) from a total of approximately 400 students. In order to select the two groups, grades in plane geometry, physics, general science and elementary algebra were utilized.

The grades were in the form of A,B,C,D,E and F. Scores of 6,5,4,3,2 and 1 were respectively assigned to the grades. A grade point average of 4.75 was used as the lower limit for the definition of the high group, while a grade point average of 2.5 was used as the upper limit for the definition of the low group. A "t" test was performed to see if the two groups differed significantly. Table 1 presents the statistical results.

Table I

	Criterion grade	N	Mean grade of the group	σ(unbiased)	"t" value	level of significance
High	4.75	35	5.292	.38092	39.56	.001
Low	2.50	35	2.228	.25980		

C) Description of the problems used.

The technique to be used in this study was originally employed to explore the mental processes involved in medical diagnosis. Later on the method was adapted for use in other fields, as it has been mentioned in the review of the related research. The nature and the construction of the problems used in this type of research have been extensively described in previous publications (27,32,33). At the beginning the problems were scored using group norms. Later the schemata norms were developed. "In essence this approach consists in specifying the set of logical relationships that one desires to include in a problem. These "logical frames" or "schemata" can be graded in terms of the number and type of relationships involved, and an ordinal scale of difficulty can be developed which provides a way of evaluating the "intrinsic" difficulty of each problem, and some of its properties, before administering it to the subjects."

"Different types of contents of varying familiarity can be used on the same schema to construct a problem. Thus, a given set of logical relationships can be expressed in terms of concrete or symbolic language at different degrees of abstraction. A set of problems can be constructed, different in terms of the language in which they are stated but identical in terms of the logical relationships involved. The relationships used in most of the problems can be constructed, different in terms of the language in which they are stated but identical in terms of the logical relationships involved. The relationships used in most of the problems built for this research were modeled after those found in some classical mathematical concepts i.e. combinatorial theory, simultaneous equation, etc". (Rimoldi, 1963)

In terms of the administration, the subject is presented a problem which is printed on a three-by-five index card. The questions are also presented on similar cards. One question per card with the corresponding answer on the reverse side. The problem itself is given on the first card. In order to ask the question, the subject picks up a card and after reading it, keeps it on one side, so that he can refer back to the information at a later stage if he wants it.

The subject is asked to read the problem carefully and then to read all the questions. After this he is asked to pick up a question, the answer to which he wants last. He then picks up the card, turns it over for the answer. He keeps on doing this with other cards until he feels that he has enough information to offer a solution to the problem. (The copies of the problems are given in the appendix).

D) Scoring methods

1) Number of questions asked

The number of questions asked by a subject or group of subjects can be used as a measure of the thought processes involved in problem solving. The precise meaning given to this measure may vary according to the particular experimental situation. One possible interpretation of this measure is that the number of questions asked reflects the relative efficiency by which the subjects are able to solve a particular problem. In terms of this interpretation, it is expected that good problem solvers will ask fewer questions than poor problem solvers. Strictly speaking, however, this measure cannot be used in an absolute manner to characterize the quality of thought process since it may be the product of other variables such as guessing.

2) Correct solutions

The number of correct solutions for a given problem can be used as a measure to evaluate the performance of a group. The underlying assumption for this particular scoring procedure is that the group with high mathematical ability would be expected to get a greater number of correct solutions than a group with low mathematical ability. However, this assumption may be a suspect because the correct solution might be a matter of chance or guessing rather than mathematical ability. Involved with this is the assumption that the problem is a good discriminator between two groups. That is, if a problem is too easy or too complex for a particular group to solve, analysis in terms of correct solutions will not be adequate.

3) Group norms

Rimoldi (1960) divised a technique in which the subjects could be scored in terms of group norms. This technique utilizes the frequency of the selection of a particular question in a particular order. For a given group, a table can be constructed (using either frequencies or proportions) which specifies the selection of all the questions asked by the group in their specific order. Each subject, then, can be scored cumulating the values for each question asked. This method takes into account the order in which a question is asked; also, all questions chosen will receive some value. A subject's score using this method is essentially a reflection of his agreement with the group used to construct the scoring table.

4) Ellipsoids and Parallelograms

By means of the group norms, the individuals are scored on their unique performances. There is another technique devised by Rimoldi which has been presented in several papers (Rimoldi, Devane, Haley, 1959; Rimoldi, Devane, 1961) which enables a particular group to be evaluated. "This is done by determining the frequency with which each question has been asked by the group. Then a ratio is found between this number and the total number of questions asked by the group. This was defined as the utility index. (Rimoldi, 1955). If the questions are ranked in terms of their utility indices and cumulated for successive steps, a maximum curve (when cumulating from higher to lower utility indices) and a minimum curve (when cumulating from lower to higher utility indices) will result. Between maximum and minimum curve an ellipsoid is generated. The area of this ellipsoid will vary with the problems and the groups. If all the questions have the same utility indices, the ellipsoids degenerate into a straight line with slope proportional to the constant utility index. In the case in which some of the questions are asked by all the subjects (utility index = 1.00) and the remaining by no subject (utility index = 0) the ellipsoid becomes a parallelogram. From this it can be inferred that the area of the ellipsoid is partially a function of the dispersion of the values of the utility indices, i.e., agreement of the subjects on the usefulness of the questions asked."

"The ratio between the area of the ellipsoid and the parallelogram may be taken as an indication of the amount of agreement among the subjects concerning the utility of each question, in relation to perfect agreement". (Rimoldi, Devane, Haley, 1961; Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharia, 1962)

5) Schemata norms

Besides scoring the subjects in terms of group norms, a technique was devised which was not based upon the performance of the group but rather on the properties of the problem itself (Rimoldi, Haley, Fogliatto, and Erdmann, 1963). The frequency of selection of each question in a particular order is determined on the basis of the logical relationship involved. As in the case of group norms, these frequencies are then converted to proportions to indicate the percentage of total possible selections of that particular question in that particular order.

As opposed to the case of group norms, questions not required by the logical structure of the problem would have been a score of zero. Irrelevant questions will get scores of zero as will the relevant questions asked at the wrong time in a sequence. Therefore, this scoring method evaluates the subject, not in terms of his agreement with the performance of a particular group, but in terms of the logical structure of the problem.

6) Plateaux

As already mentioned above, an irrelevant question or a question not asked in the right order of the sequence would get a score of zero using the schemata norm. If a subject's performance curve is drawn, cumulating his scores according to the schemata norms, such questions will evidence themselves as plateaux. A score of zero would thus mean in some sense or another, the subject's inability to grasp the logical relationship at all, or not at the proper time. So an analysis of this may throw light on the process involved in solving that particular problem. The mean number of plateaux for a given problem can be calculated for a particular group. This supplies an index of the understanding of the logical structure of this particular problem by this particular group.

7) Convex sets

Convex sets have been discussed in several articles. (Rimoldi, Haley, 1962; Rimoldi, Fogliatto, Haley, Reyes, Erdmann, Zacharia, 1962; Fogliatto, Haley, Reyes, Erdmann, Zacharia, 1962; Fogliatto, 1963; Riedel, 1963). Essentially, a convex set is constructed by plotting. For each subject in a given group, the observed score for a given sequence of questions on the abscissa and the observed minus the expected score on the ordinate, the set is drawn by joining the outermost points of the plot.

The sets can be drawn using any scoring system available. Similarly, expected scores can be obtained from any particular hypothesis one wishes to test. In the present study, both group norms and schemata norms were used as the observed scores; and, a table of random values were used as the expected scores. The quality of each subject's process is indicated by its position in the convex set. A particular tactic's position may or may not vary according to the particular scoring system used to determine both the observed and expected values.

CHAPTER IV

RESULTS AND DISCUSSION

A) Number of questions asked.

There is no systematic difference between the two groups using this type of analysis. Table II indicates the mean number of questions selected by each group for all the four problems. Problems 35B and 39A show a significant difference whereas problem 35A and 39B do not show a significant difference. It would seem that the number of questions is not an appropriate criterion for differentiating the two groups.

Table II

Mean number of questions for both groups on each of the four problems.

	35A	35B*	39A*	39B
High	5.8	6.2	5.5	5.4
Low	5.7	4.3	6	5.7

* P .01

B) Correct solutions

The number of subjects who solved each problem correctly was also analyzed. A Chi-square test was performed on the number of correct and incorrect solutions for each group on every problem. Table III shows the results of this analysis. Problems 35B and 39B differentiated between the high and low group at .001 level or better in the expected direction. Notice that problems 35A and 39A that are presented in a concrete verbal context fail to differentiate between the two groups. Nevertheless, the introduction of an abstract language produces clear differences in performance, in spite of the fact that the logical frames are the same.

Table III

Chi-square analysis of correct and incorrect solutions.

Problem No.	35A	35B*	39A	39B*
X^2	1.64	16.97	1.69	15.21

* Significant at .001 level

C) Group norms.

When subjects were scored according to the group norms, no significant differences were found between the two groups. Analysis in terms of group norms evidenced lower mean scores for the high group on all the problems. The differences, however, were not significant. Problem 35A approaches significance (p = .08, two failed test). These results bear out what has been found in previous work, that group norms do not necessarily act as effective discriminators. Table IV presents the mean scores for both the groups for all the problems.

However, when the subjects were scored in terms of utility indices, (Rimoldi, 1955) significant differences emerged on all the problems. The means of the low group, in almost every case, were greater than those of the high group. This indicates that the low group selected more of those questions which were "popular" in the general sample. The high group, then, conformed to a lesser extent to the performance of the general sample. Table IVa presents the mean utility index scores for both the groups for all the problems.

Table IVa

Mean Utility Indices

	35A**	35B*	39A***	39B***
High	.623	.585	.752	.680
Low	.688	.628	.822	.670

* Significant at .08 level
** Significant beyond .05 level
*** Significant beyond .01 level)

Table IV

Mean Scores based on group norms.

	35A	35B	39A	39B
High	.062207	.067137	.228670	.190291
Low	.074403	.068877	.223838	.192723

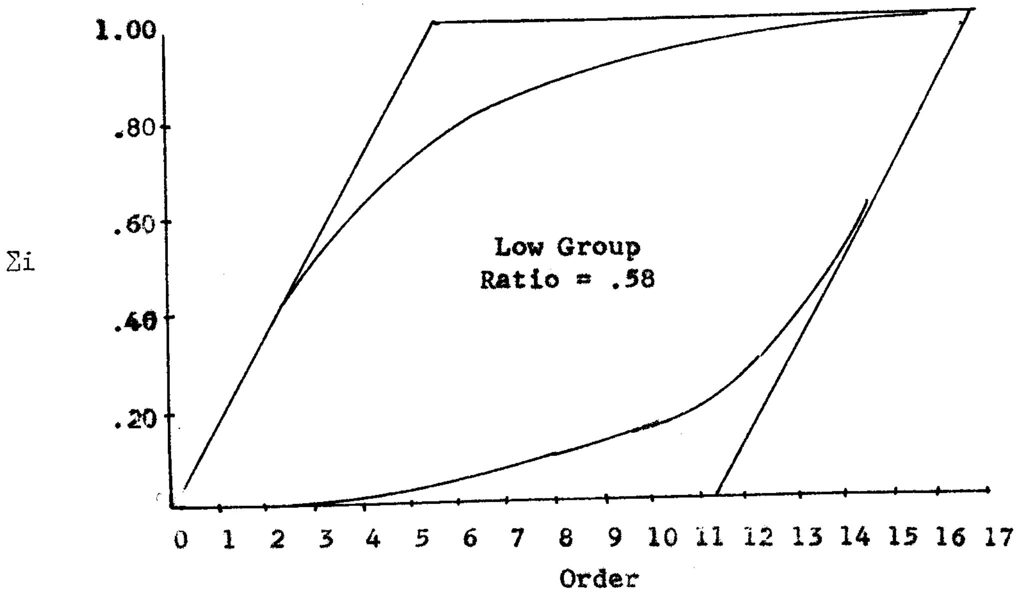
D) Ellipsoids and Parallelograms

Ellipsoids for both the high and low groups were drawn for all the problems and they are presented in figures through 1 to 4 respectively. The Kolmogorov-Smirnov test was used to compare the maximum curves of the ellipsoids for both the groups for all the four problems. The differences for the problems 35A, 35B, and 39A were in the same direction, the low group having a greater acceleration of the maximum curve than the high group. This was not so in the case of problem 39B. The differences for problems 35A, 35B, 39A, and 39B were significant at .05, .10, .20, and .50 levels respectively. More rapid acceleration of the maximum curve can be interpreted as evidencing greater homogeneity of performance. Also, greater ratio between the ellipsoid and the parallelogram can be interpreted as an indication of the amount of agreement among the subjects concerning the utility of each question in relation to perfect agreement. Thus, the high group was more variable within itself than the low group. Table V presents the ratios between the ellipsoids and the parallelograms and the results of the Kolmogorov-Smirnov test for the maximum curves for all the four problems.

Table V

The ratios between the ellipsoids and the parallelograms and the results of the Kolmogorov-Smirnov test for the maximum curves.

	Ratio			
Problems	High	Low	X^2 Kolmogorov-Smirnov	P
35A	.48	.58	6.5	$<.05$
35B	.51	.53	5.7	$<.10$
39A	.57	.68	4.5	$<.20$
39B	.57	.50	2.2	$<.50$



Chi Square = 6.5+

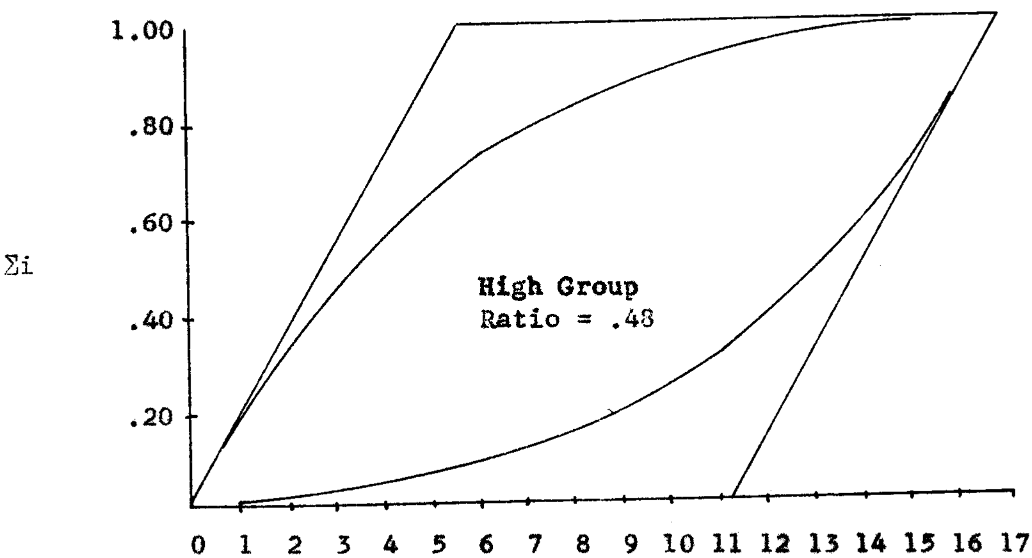
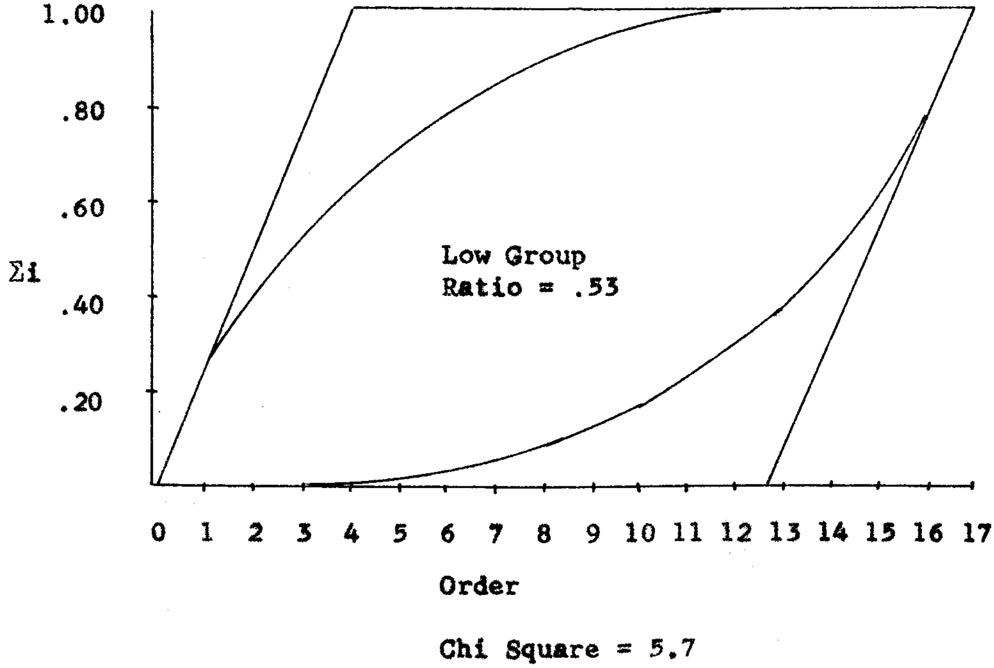


FIGURE 1

ELLIPSOIDS AND PARALLELOGRAMS GENERATED BY THE PERFORMANCE OF HIGH AND LOW GROUPS FOR PROBLEM 35A.



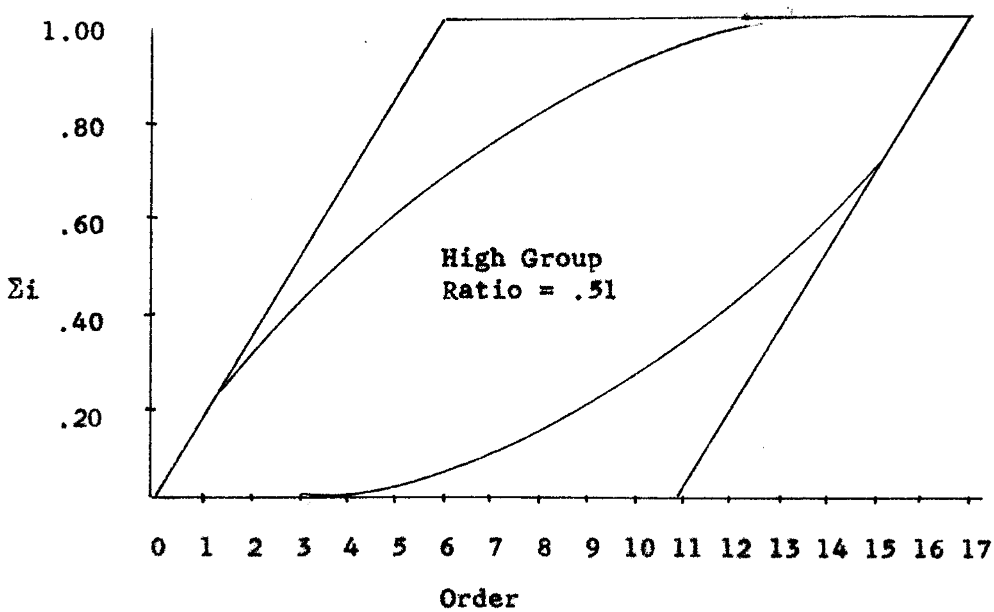
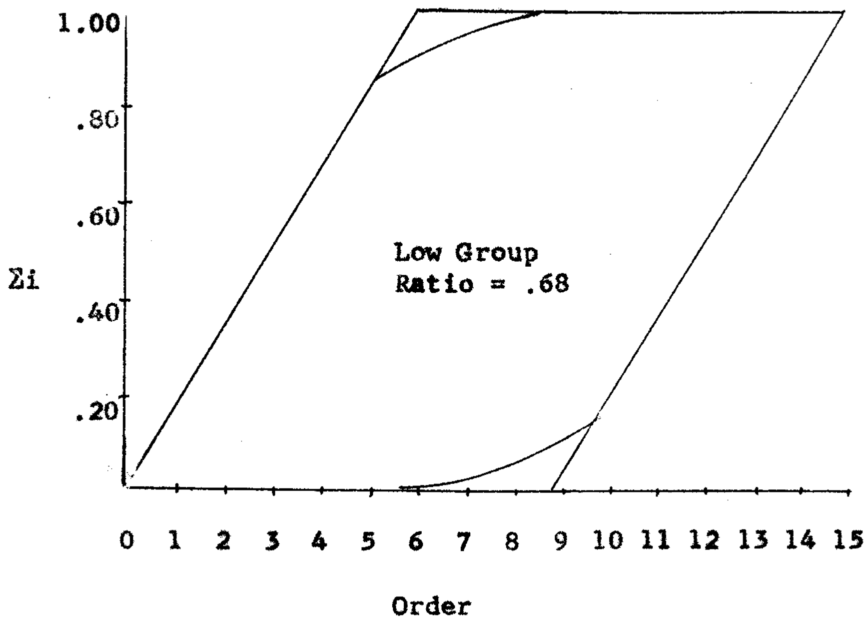


FIGURE 2

ELLIPSOIDS AND PARALLELOGRAMS GENERATED BY THE PERFORMANCE OF HIGH AND LOW GROUPS FOR PROBLEM 35B



Chi Square = 4.5

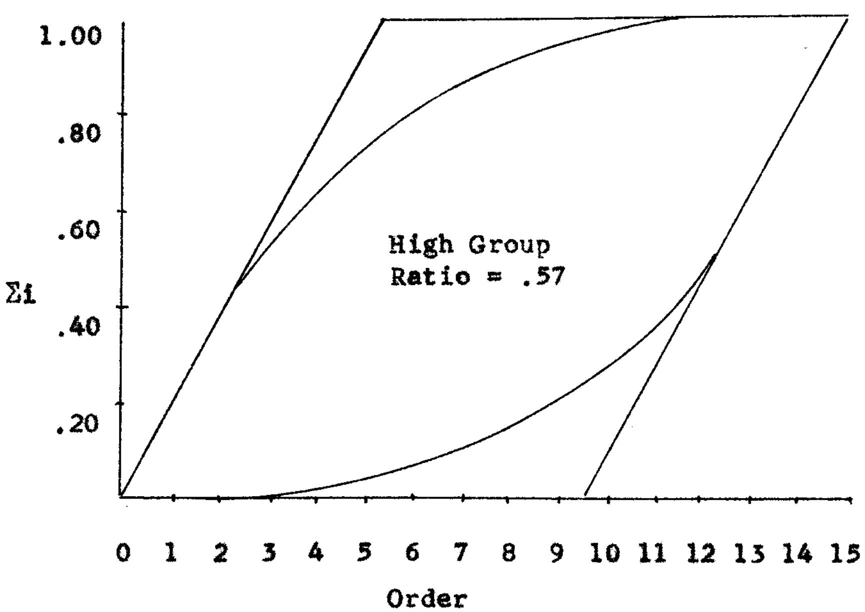
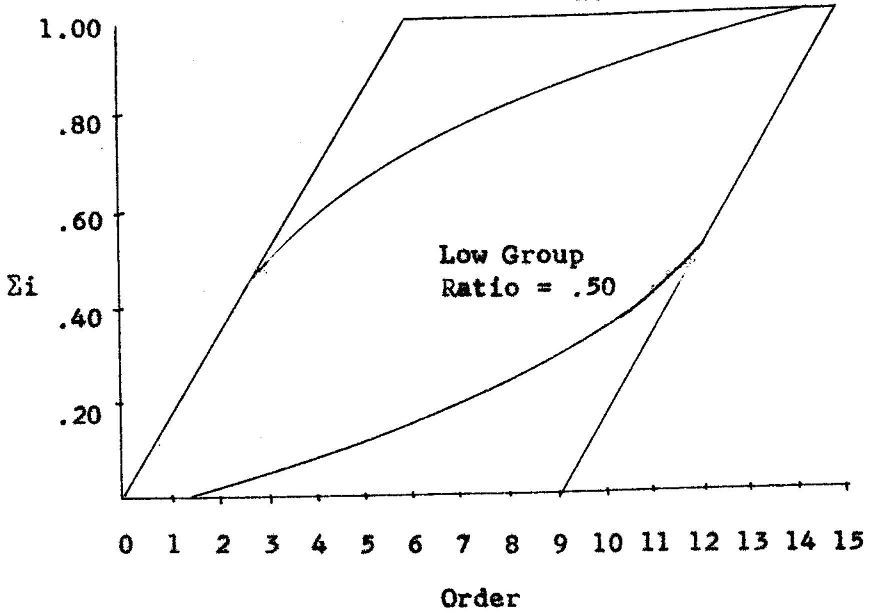


FIGURE 3

ELLIPSOIDS AND PARALLELOGRAMS GENERATED BY THE PERFORMANCE OF HIGH AND LOW GROUPS FOR PROBLEM 39A



Chi Square = 2.2

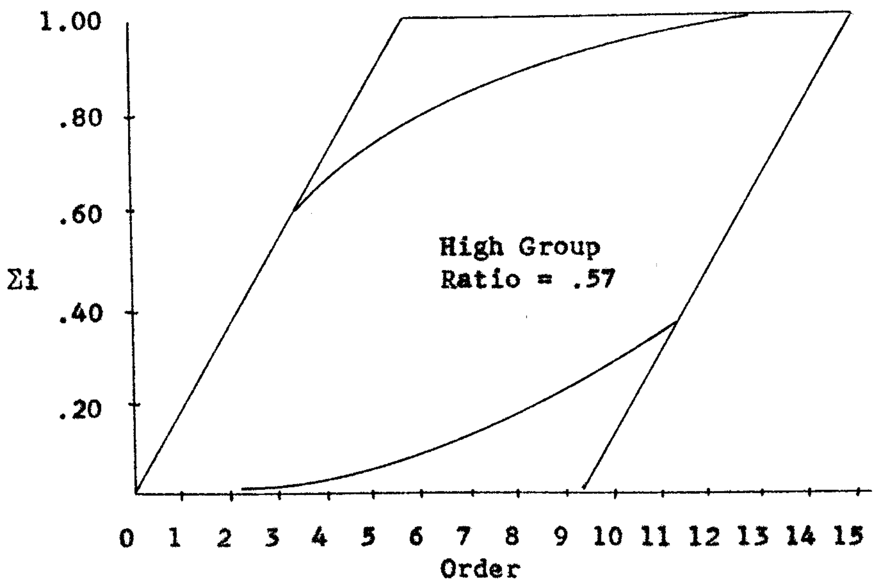


FIGURE 4

ELLIPSOIDS AND PARALLELOGRAMS GENERATED BY THE PERFORMANCE OF HIGH AND LOW GROUPS FOR PROBLEM 39B

E) Schemata norms

When problems 35A and 35B were scored according to the schemata norms, it was found that the two groups were not differentiated. The mean scores for both the groups for 35A and 35B are given in Table VI. The mean score for the low group is higher than the mean score of the high group for problem 35A. The mean score for problem 35B is identical for both the groups.

Table VI

Mean scores according to the schemata norms

	35A	35B
High	.0709	.0806
Low	.0852	.0806

Problems 39A and 39B did not readily lend themselves into a clear-cut interpretation according to schemata norms. Hence, these problems were not scored according to them.

F) Plateaux

The performance curves for all the subjects according to the schemata norms were drawn for problems 35A and 35B. Due to space limitation, it is impossible to include all the curves in this study. Figure 5 presents the performance curve for one subject of high and one subject of low "mathematical ability" for problems 35A and 35B. By inspection of this figure, it can be seen that there is a moment in the curve when no increment is shown, which means that the subject has selected an irrelevant question or has asked a relevent question in the wrong order. This plateaux can appear at any moment in the performance curve.

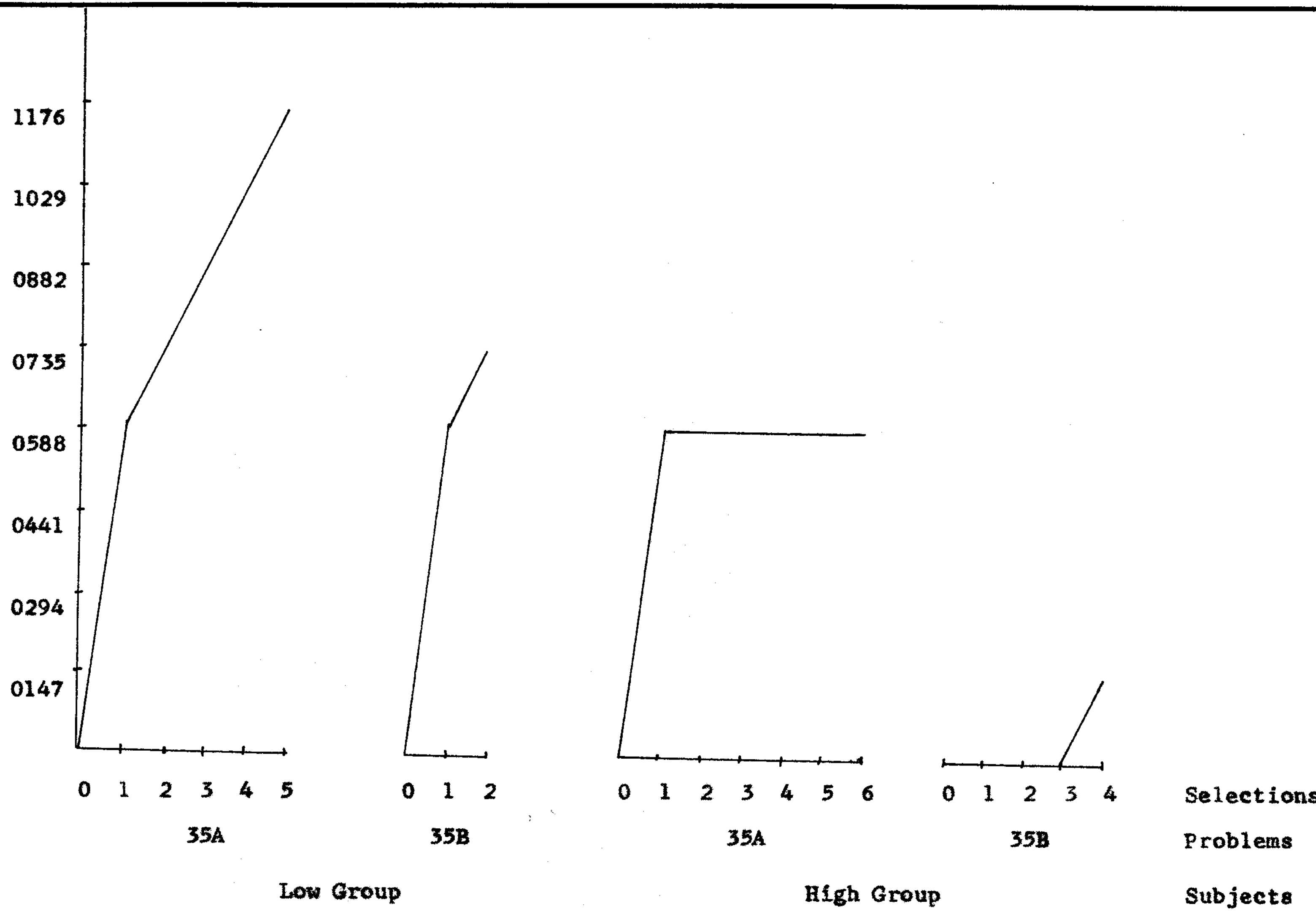


FIGURE 5

PERFORMANCE CURVES FOR A SUBJECT OF THE LOW GROUP AND A SUBJECT OF THE HIGH GROUP (SCHEMATA NORMS) FOR PROBLEM 35A AND PROBLEM 35B

When an analysis of plateaux or zero responses was performed, a significant difference between the two groups for problem 35B was found. However, problem 35A did not differentiate significantly. Table VII shows the mean number of zero responses.

Table VII

	35A	35B*
High	2.80	2.80
Low	2.20	1.48

* $t = 2.27$ Significant beyond .05 level.

G) Convex sets

Polygonal convex sets were drawn by plotting the 0 (observed) score on the abscissa and 0-E (observed minus expected) on the ordinate. The sets for the both high and low groups were based on group norms (35A, 35B, 39A, 39B) and schemata norms (35A, 35B).

Figures 6 to 11 inclusive present the polygonal convex sets with the subjects designated with the tactics that they followed. For the convex sets based on the schemata norms, subjects placed in the upper right hand corner are the ones who followed the most logical tactic. For the convex sets based on the group norms, subjects placed in the right hand corner are the ones who follow the most popular tactic of the group.

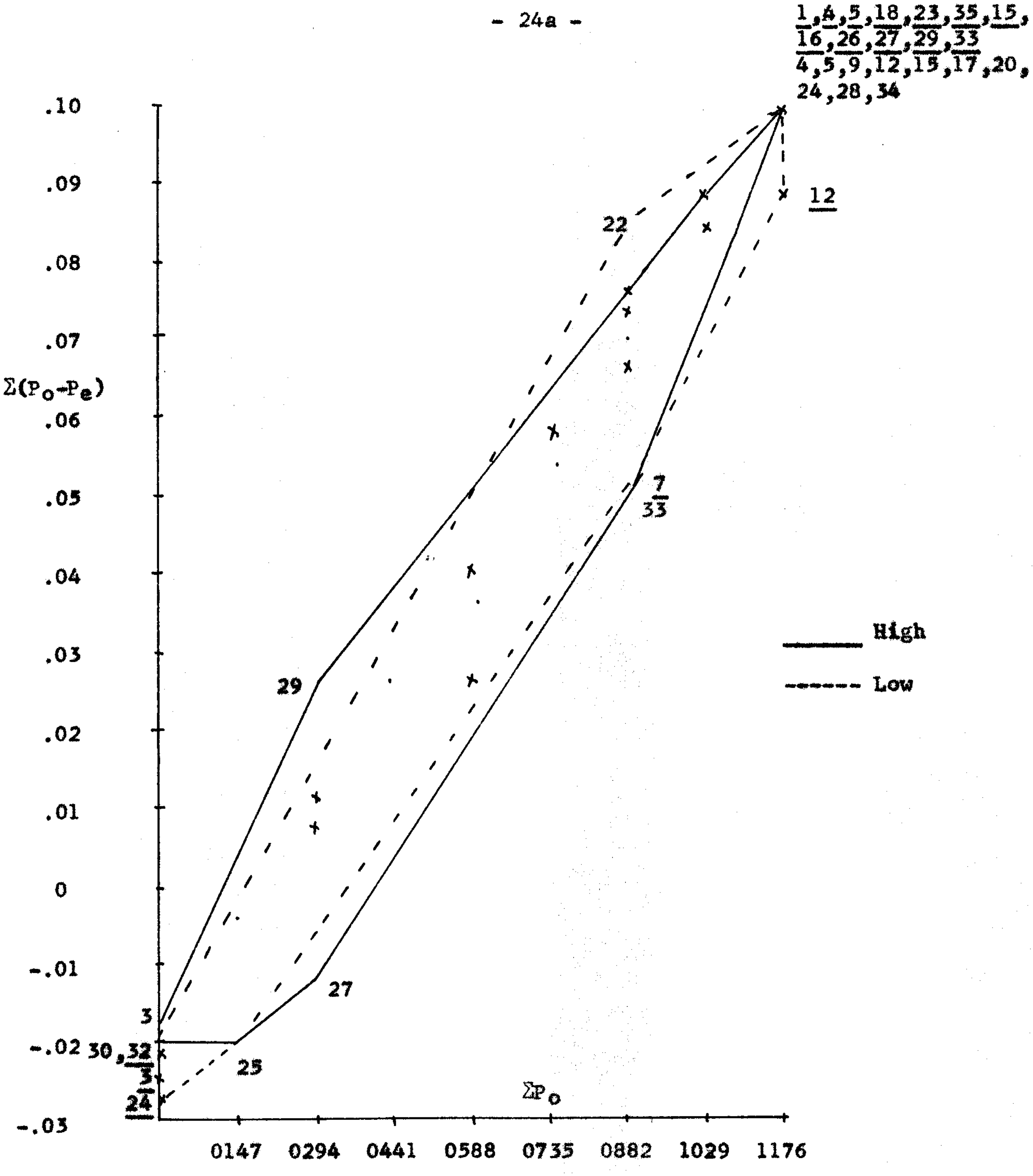


FIGURE 6

LIMITS OF PERFORMANCE OF HIGH AND LOW GROUPS ON PROBLEM 35A BASED ON SCHEMATA NORMS.

Tactics Corresponding to FIGURE 6

High Group		Low Group	
Subjects	**Tactics**	**Subjects**	**Tactics**
34	4,6,2,7,15	1,4,5,18,23,35	4,2,6,7,15
20	4,10,14,3,16	15,16,26,27,29	4,2,6,15,7
12	4,2,6,15,7	33	4,6,2,7,15
9,15,17,24	4,2,6,7,15	12	4,10,14,7,16,6,2,5
5	4,14,10,15,3	7	4,7,15,3,16,10,17, 14,13,11
4	4,10,14,16,3		
28	4,6,2,15,7	24	7,15,4,10,13,16,14,3
33	4,7,15,3,16,9,13, 11,17,10,14	3	1,4,3,6,2,7,10
		32	9,15,7,4,2,6
27	3,4,6,7,9,10,11,13, 14,15,16,17	22	4,6,2,7,5
25	1,7,15,4,3,9,17,14, 10,16		
30	1,7,15,4,6,2		
3	7,15,4,2,6		
29	15,4,2,6,7		

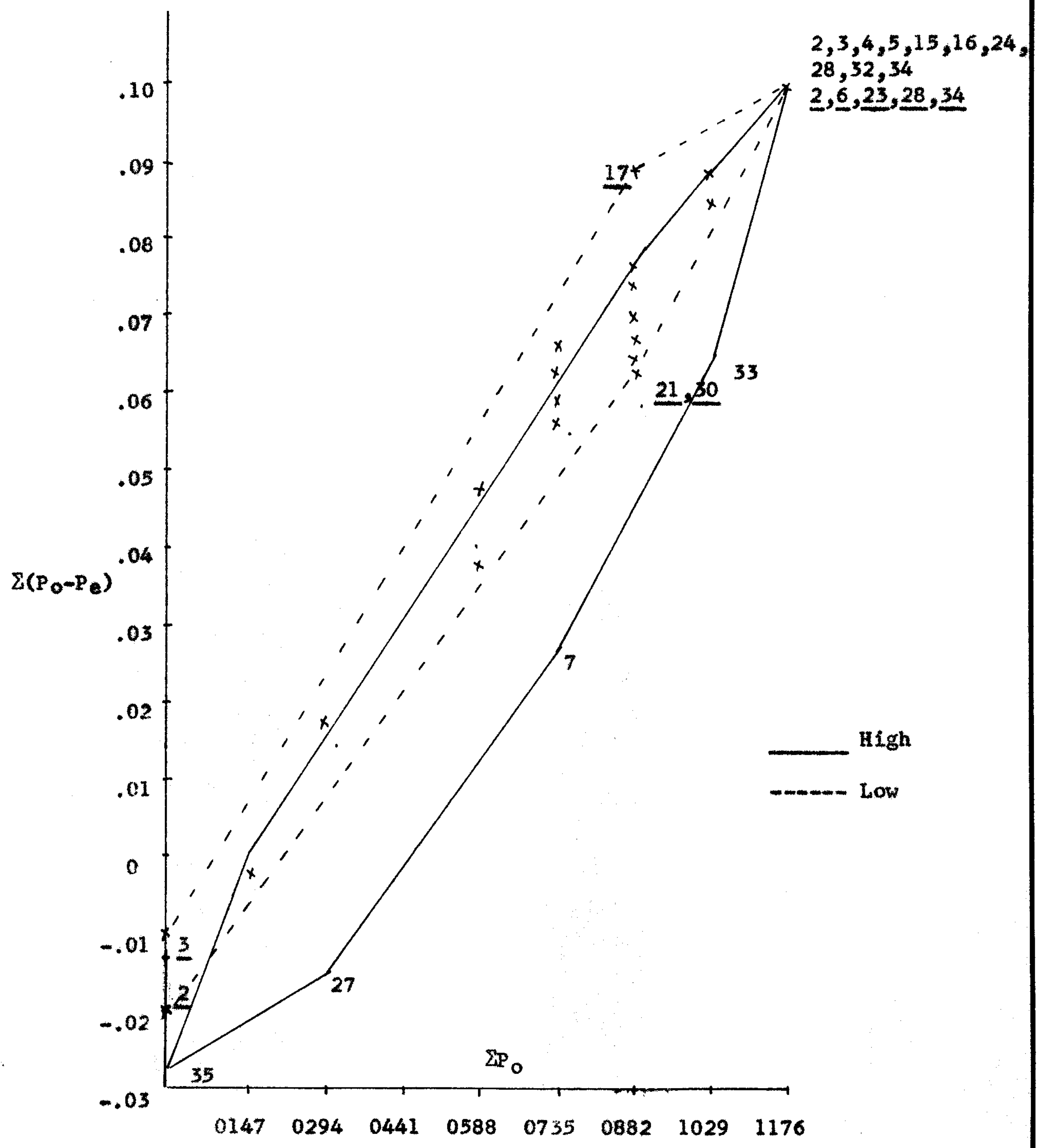


FIGURE 7

LIMITS OF PERFORMANCE OF HIGH AND LOW GROUPS ON PROBLEM 35B BASED ON SCHEMATA NORMS

Tactics Corresponding to FIGURE 7

High Group Subjects	High Group Tactics	Low Group Subjects	Low Group Tactics
2,5,15,32	5,6,15,4,13	2	5,6,15,10,4
3,16,24,34	5,15,6,4,13	6	5,15,6,4,10
4	5,7,2,10,14	23	5,6,15,13,4
28	5,15,6,13,4	28	5,15,6,14,10
33	5,6,2,4,7,10,15,8,	34	5,6,15,10,14
	14,13,11	21	5,6,2,7,3,10,8
7	5,10,9,8,16,14,12,	30	5,7,13,6,10,9,4
	13,4	12	15,14,10,11,6,4
27	2,6,7,15,5,4,14,16,	3	14,4,8
	13,12,10,9,8	17	5,2,7
35	3,4,5,6,7,8,9,13		
1	1,4,10,14		

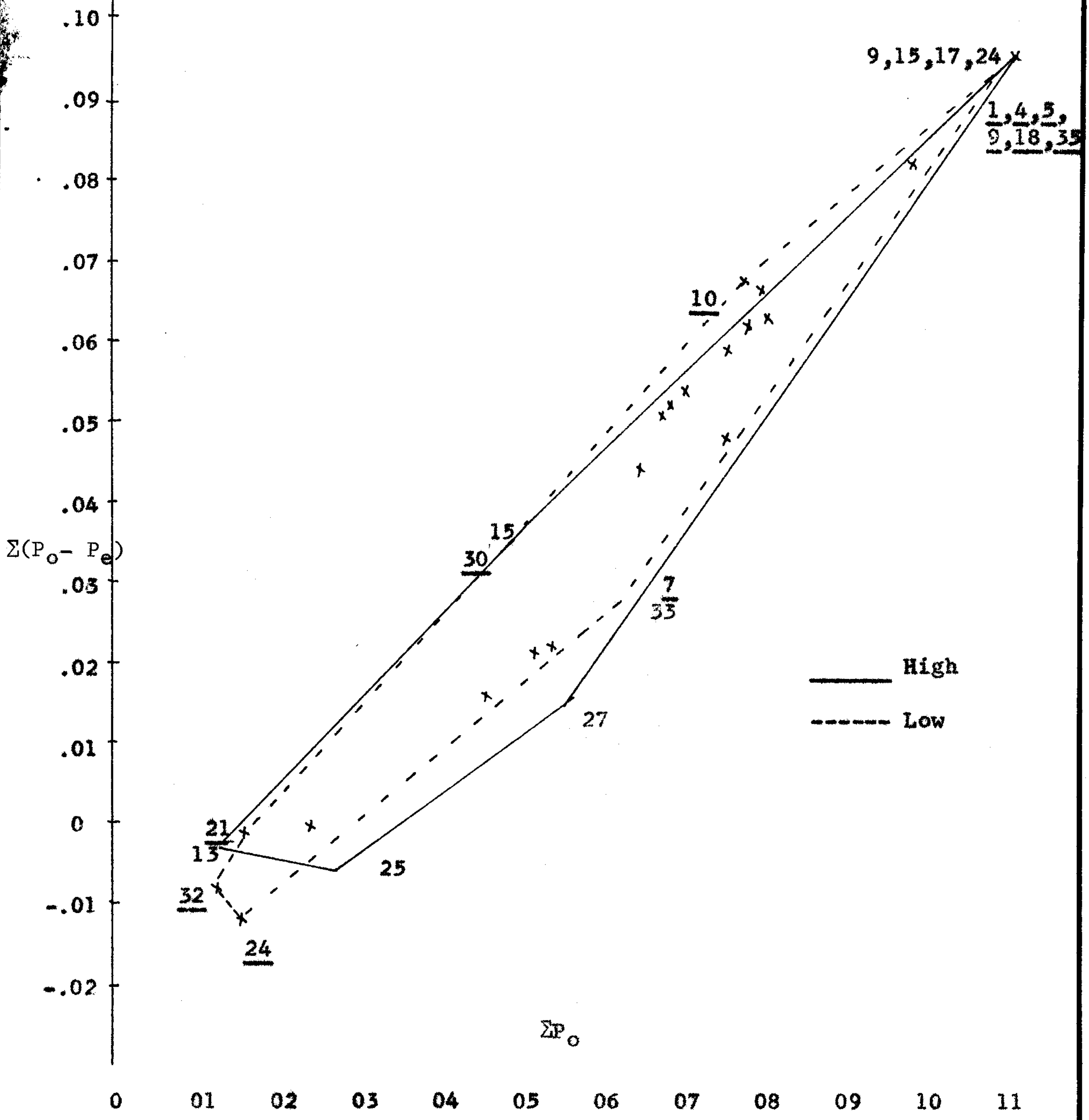


FIGURE 8

LIMITS OF PERFORMANCE OF HIGH AND LOW GROUPS ON PROBLEM 35A BASED ON GROUP NORMS

Tactics Corresponding to FIGURE 8

High Group

Subjects	Tactics
9,15,17,24	4,2,6,7,15
33	4,7,15,3,16,9,13, 11,17,10,14
27	3,4,6,7,9,10,11, 13,14,15,16,17
25	1,7,15,4,3,9,17, 14,10,16
13	7,15,10,4,2
15	4,14,10,16

Low Group

Subjects	Tactics
1,4,5,9,18,15	4,2,6,7,15
7	4,7,15,3,16,10,17, 14,13,11
24	7,15,4,10,13,16,14,3
32	9,15,7,4,2,6
21	7,15,10,2,6
30	4,15,2,6
10	4,2,6

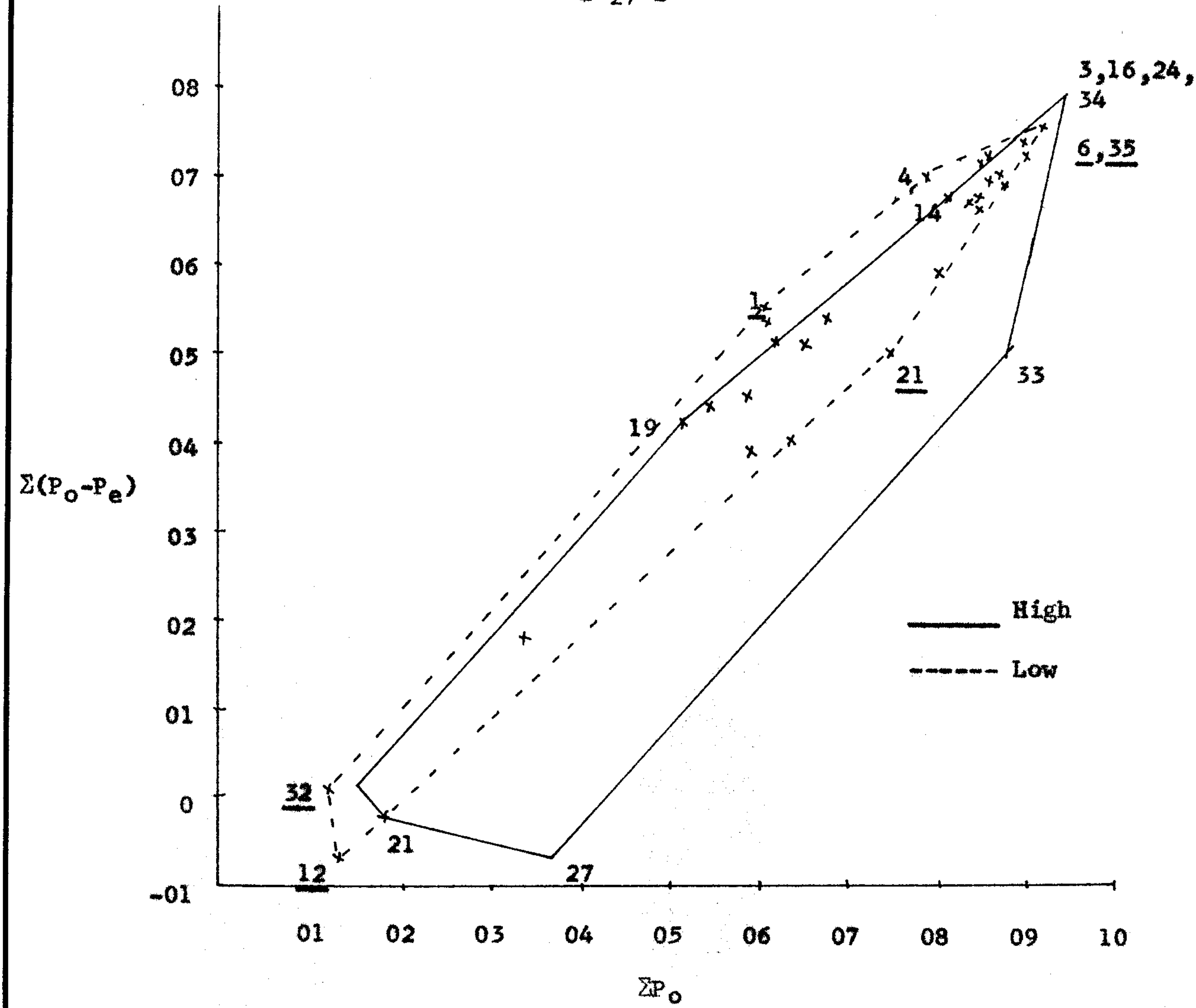


FIGURE 9

LIMITS OF PERFORMANCE OF HIGH AND LOW GROUPS
FOR PROBLEM 35B BASED ON GROUP NORMS

High Group

Subjects	Tactics
3,16,24,34	5,15,6,4,13
33	5,6,2,4,7,10,15,8,14, 13,11
27	2,6,7,15,5,4,14,16, 13,12,10,9,8
21	4,13,10,14,12,16
1	1,4,10,14
19	5,7,2
14	5,6,15,10

Low Group

Subjects	Tactics
6	5,15,6,4,10
35	5,15,6,2,7
21	5,6,2,7,3,10,8
12	15,14,10,11,6,4
32	4,2,7
1	5,15
4	5,15,6

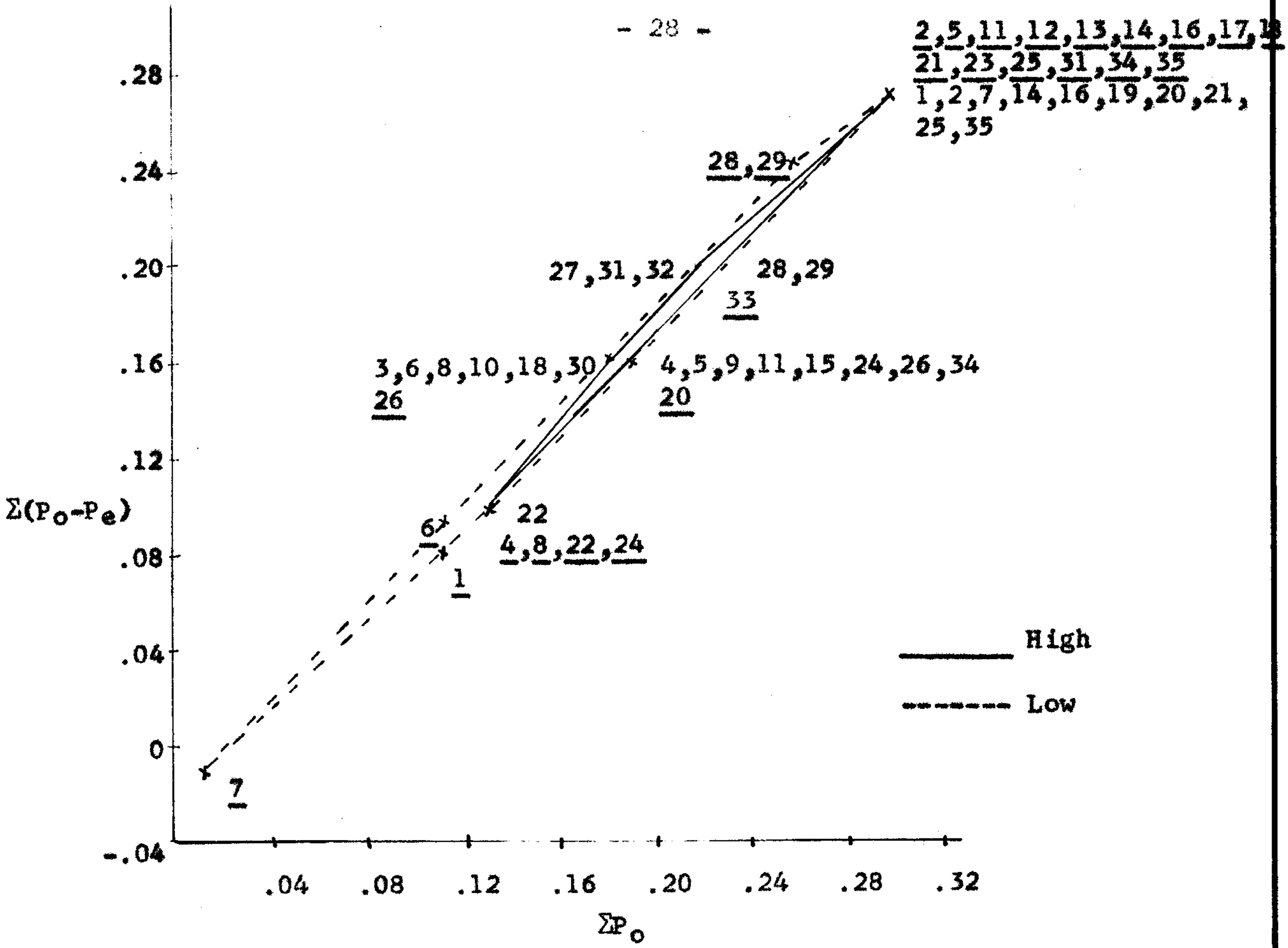


FIGURE 10

LIMITS OF PERFORMANCE OF HIGH AND LOW GROUPS ON PROBLEM 39A BASED ON GROUP NORMS

High Group		Low Group	
Subjects	**Tactics**	**Subjects**	**Tactics**
1,2,7,14,16,19,20, 21,25,35	1,2,3,4,7,13	2,5,11,12,13,14, 16,17,18,21,23,25, 31,34,35	1,2,3,4,7,13
28	1,2,3,4,13,7	33	1,2,3,4,5,6,15,9
29	1,2,3,7,4,13	20	1,2,3,5,6,4
4,9,15,24,26,34	1,2,3,5,15	4,8,22,24	1,4,2,7,3,13
5,11	1,2,3,5,6	1	3,2,1,4,13,7
22	1,4,2,7,3,13	7	2,1,12,7,3
3	1,2,3,15,6	6	1,3,2,13,7
6,18	1,2,3,14,8	26	1,2,3,6,9
8,10	1,2,3,6,9	28	1,2,3,4,7,15
30	1,2,3,9,6	29	1,2,3,4,7,12
27	1,2,3,8,4,13		
31	1,2,3,13,7,4		
32	1,2,3,6,7		

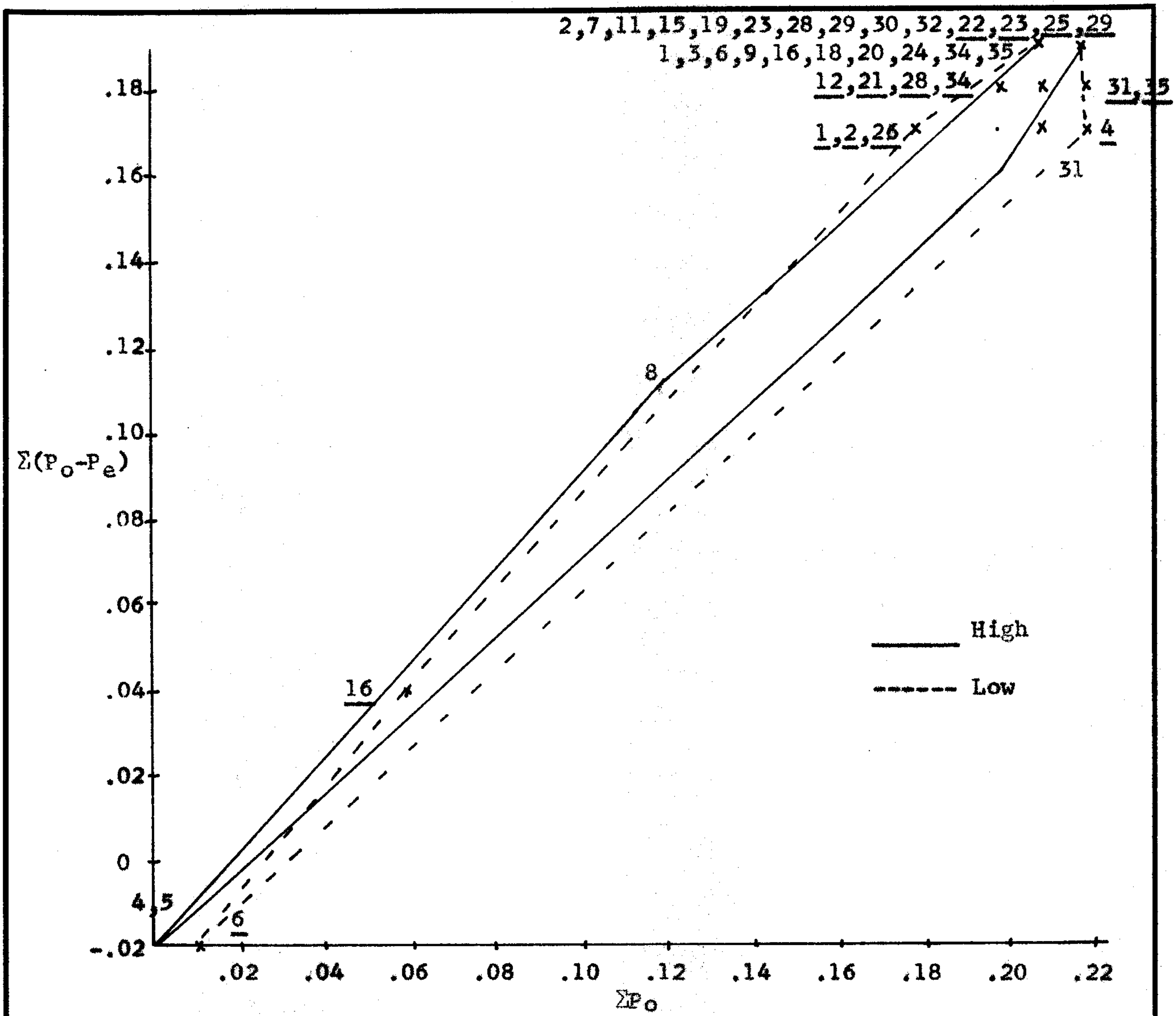


FIGURE 11

LIMITS OF PERFORMANCE OF HIGH AND LOW GROUPS ON PROBLEM 39B BASED ON GROUP NORMS

High Group		Low Group	
Subjects	Tactics	Subjects	Tactics
11,15,19,29,32	1,2,3,5,6	22,23	1,2,3,5,6,9
23	1,2,3,4,5,6	25	1,2,3,5,6,7
28	1,2,3,5,14,15	29	1,2,3,4,7,13
2,7,30	1,2,3,4,7,13	31,35	1,2,3,5,6,8,9,14,15
31	1,2,3,13,5,7,6,9	4	1,2,3,4,5,6,7,8,9,10
4	4,7,14,1,2,3	6	9,7,1,2,3,10
5	4,5,6,12,2,3	16	1,3,2,15
8	1,2,6,9	1,2,26	1,2,3
1,6,9,18,20,34,35	1,2,3,5,15	7	1,2,3,5
3,16,24	1,2,3,5,9	12,21,28	1,2,3,5,15
		34	1,2,3,5,13

CHAPTER V

SUMMARY

The purpose of this study was to investigate the relationship between problem solving processes and mathematical ability as defined by grades in school. Previous studies in this area have focused upon product, rather than process, as this study does. The method developed by Rimoldi was utilized to study the set of decisions involved in problem solving rather than just the final solution.

The subjects used in this study were from Chicago area high school (St. Rita). A sample of 70 students was divided into two sub-groups (each consisting of 35 subjects) on the basis of their grades in mathematics and science (Elementary Algebra, plane geometry, physics and general science). Grade point averages were used for the definition of two groups, the values 6,5,4,3,2 and 1 being arbitrarily assigned to grades A,B,C,D,E and F respectively. The high group was selected on the basis of having a grade point average of 4.75 or above. The low group was selected on the basis of having a grade point average of 2.5 and below.

All the subjects were administered four problems, i.e., 35A, 35B, 39A and 29B. The performance of all the subjects was examined in terms of group norms, schemata norms, number of questions asked, correct answers, and plateaux in the performance curves. In addition; convex sets, ellipsoids and parallelograms were used to describe and interpret the results. Analyzing problem solving in this manner allows both conventional analysis of right and wrong answers and analysis of the processes or tactics used by the subject to reach the final solution.

In terms of the number of questions asked, problems 35B and 39A differentiated at .01 level of significance. The other two did not. Evaluation of the two groups in terms of correct solutions evidenced a significant difference at the .01 level for problems 35B and 39B.

Analysis in terms of group norms yielded lower mean scores for high group on all the four problems. However, the difference between the two groups was not significant.

When the subjects were scored in terms of utility indices, significant differences emerged on all the problems. The means of the lower group were greater on all the problems.

In order to analyze group performance ellipsoids were drawn for both groups and were compared using the Kolmogorov-Smirnov test for significant differences. The differences for all the problems except 39B were in the same direction, the curve of the low group having a greater acceleration than that of the high group. The differences for problems 35A, 35B, 39A and 39B were significant at .05, .10, .20 and .50 levels respectively. The ratios between the ellipsoids and the parallelograms for the high group for problems 35A, 35B, 39A, 39B were .48, .51, .57, .57 and for the low group were .58, .53, .68, and .50 respectively.

According to the schemata norms there was no significant difference on either 35A or 35B. The analysis of plateaux showed a significant difference at .05 level in the case of problem 35B, though not in the case of 35A.

The convex sets based on the group norms (35A, 35B, 39A, 39B) and schemata norms (35A, 35B) were drawn.

As can be seen in this study, the use of the technique developed by Rimoldi yields much more information concerning problem solving process than more conventional techniques. The additional information indicates that much more is involved than the correctness or incorrectness of the final answer.

CHAPTER VI

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APPENDIX

Table I *

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem No. 35A

Questions

Order	2	3	4	6	7	10	14	15	16
1.			0588						
2.	0147			0147		0147	0147		
3.	0147			0147		0147	0147		
4.		0147			0147			0147	0147
5.		0147			0147			0147	0147

* All questions which do not appear in this Table receive a score of zero.

Table II *

Values Corresponding to Each Group According to the Scoring Method Based on the Schemata for Problem No. 35B

Questions

Order	2	4	5	6	7	10	13	14	15
1.			0588						
2.	0147			0147	0147				0147
3.	0147			0147	0147				0147
4.		0147				0147	0147	0147	
5.		0147				0147	0147	0147	

* All questions which do not appear in this table receive a score of zero.

PROBLEM 35A

Instructions and Corresponding Questions and Answers

A college choral group is composed of freshmen, sophomores and juniors. The chorus has three voices or parts which are high, medium, and low. The questions and answers below give vital information concerning the group. From these facts you are to find the number of juniors singing the middle or medium part.

Questions		Answers	
1.	How many Juniors are in this college?	1.	1567
2.	How many Freshmen are in the chorus?	2.	23
3.	How many Sophomores are in the middle voice?	3.	10
4.	How many chorus members are there?	4.	76
5.	How many girls are in the chorus?	5.	45
6.	How many sophomores are in the chorus?	6.	28
7.	How many juniors sing the high voice?	7.	7
8.	How many freshmen are in this college?	8.	1848
9.	How many freshmen sing the high voice?	9.	8
10.	How many low voice members are there?	10.	28
11.	How many sophomores sing the high part?	11.	9
12.	How many pianos does the chorus have?	12.	3
13.	How many freshmen sing the low voice?	13.	9
14.	How many chorus members sing the high voice?	14.	24
15.	How many juniors are in the low voice section?	15.	10
16.	How many freshmen sing the middle voice?	16.	6
17.	How many sophomores sing the low part?	17.	9

Solution: 8 juniors

PROBLEM 35B

Instructions and Corresponding Questions and Answers

T objects are composed of M N, and P types. Each of these latter three types may or may not also be Q's R's and S's. From the questions and answers you can discover the various relationships of these objects. Make use of this available information to determine how many T objects are N's and also S's.

Questions		Answers	
1.	How many S's are A's?	1.	350
2.	How many Q's are there among the T's?	2.	19
3.	How many G's are there among the T's?	3.	43
4.	How many R's are also N's?	4.	8
5.	What is the total number of T objects?	5.	63
6.	How many P's are there among the T's?	6.	21
7.	How many R's are there among the T's?	7.	24
8.	How many Q's are also M's?	8.	5
9.	How many R's are also M's?	9.	10
10.	How many S's are also M's?	10.	2
11.	How many Q's are A's?	11.	400
12.	How many R's are also P's?	12.	6
13.	How many Q's are also N's?	13.	3
14.	How many S's are also P's?	14.	4
15.	How many M's are among the T's?	15.	17
16.	How many Q's are also P's?	16.	11
17.	How many H's among the A's?	17.	2

Solution: 14 T objects are N's and also S's

Problem 39A

Instructions and Corresponding Questions and Asnwers

A man wished to buy an automobile. After investigating all of the makes available he found three types of automobiles, A, B, and C, that were suited to his needs. In choosing among these three he decided to buy the one that would cost the least. Which type of automobile did he buy, A, B, or C?

Questions	Answers
1. What was the total cost of car A?	1. 2000 pounds
2. What was the total cost of car B?	2. 8000 francs
3. What was the total cost of car C?	3. 250,000 pesos
4. What is the value of a pound in United States currency?	4. $2.50
5. What is the value of a pound in French currency?	5. 10 francs
6. What is the value of a pound in Spanish currency?	6. 250 pesos
7. What is the value of a franc in United States currency?	7. $ .25
8. What is the value of a franc in English currency?	8. .1 pound
9. What is the value of a franc in Spanish currency?	9. 25 pesos
10. What is the value of a dollar in English currency?	10. .4 pound
11. What is the value of a dollar in French currency?	11. 4 francs
12. What is the value of a dollar in Spanish currency?	12. 100 pesos
13. What is the value of a peso in United States currency?	13. $ .01
14. What is the value of a peso in English currency?	14. .004 pound
15. What is the value of a peso in French currency?	15. .04 franc

Solution: Car B

Problem 39B

Instructions and Corresponding Questions and Answers

There are three objects, X, Y, and Z. Each object has a value that can be stated in terms of a, b, c, or d. From the following questions select those that you consider necessary to determine which of the three objects has the smallest value.

Questions	Answers
1. What is the value of X?	1. 100a
2. What is the value of Y?	2. 400b
3. What is the value of Z?	3. 12,500d
4. What is the value of "a" stated in terms of "c"?	4. 2.5c
5. What is the value of "a" stated in terms of "b"?	5. 10b
6. What is the value of "a" stated in terms of "d"?	6. 250d
7. What is the value of "b" stated in terms of "c"?	7. .25c
8. What is the value of "b" stated in terms of "a"?	8. .1a
9. What is the value of "b" stated in terms of "d"?	9. 25d
10. What is the value of "c" stated in terms of "a"?	10. .4a
11. What is the value of "c" stated in terms of "b"?	11. 4b
12. What is the value of "c" stated in terms of "d"?	12. 100d
13. What is the value of "d" stated in terms of "c"?	13. .01c
14. What is the value of "d" stated in terms of "a"?	14. .004a
15. What is the value of "d" stated in terms of "b"?	15. .04b

Solution: Y

APPROVAL SHEET

The thesis submitted by Mira H. Bhandari has been read and approved by three members of the Department of Psychology.

The final copies have been examined by the director of the thesis and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the thesis is now given final approval with reference to content, form, and mechanical accuracy.

The thesis is therefore accepted in partial fulfillment of the requirements for the Degree of Master of Arts.

Dec 2/64
Date

Signature of Adviser